A SIMPLE NETWORK ANALOG APPROACH FOR THE QUASI-STATIC CHARACTERISTICS OF GENERAL LOSSY, ANISOTROPIC, LAYERED STRUCTURES

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ABSTRACT

The quasi-static parameters of multilayered planar structures consisting of lossy and/or anisotropic dielectric media are computed by utilizing a simple, versatile discrete network analog having complex (e.g., RL, RC) branches. The method is applied to compute the propagation constants, impedances, and field distribution for typical single and coupled strip structures.

INTRODUCTION

Of all the methods that have evolved over the years for the computation of the propagation characteristics and other properties of planar structures, perhaps the most direct approach is the use of the finite difference equations [e.g., 1-3]. The corresponding resistive network analog for lossless planar structures, together with simple multiport network theory, enabled Lennartson [1] to formulate a simple, yet accurate, computational procedure for the capacitance matrix elements of coupled microstrips. More recently, the remarkably efficient and versatile method of lines has also evolved from the finite difference equations for the frequency-dependent parameters of planar structures, as well as the solution of three-dimensional problems [4-6]. In this paper, Lennartson's method is extended to apply to general lossy, anisotropic multilayered structures. In addition, it is shown that planar structures with strips at different levels, as well as the effect of strip thickness, can also be included in the analysis and computations. Also, for a given structure, the charge distribution on the strips, the potential and, hence, the electric field variation everywhere can also be evaluated by using this network analog approach. This conceptually simple, direct, yet accurate, approach is intended to complement other techniques with varying degrees of complexity, accuracy, and sophistication that have evolved over the years [e.g., 4-15] for the study of single and multilayered structures.

THE NETWORK ANALOG

The quasi-static fields are the solution of Laplace's equation subject to all the boundary conditions of the structure. For the general lossy, anisotropic case, the potential ϕ in each region is a solution of

$$\nabla \cdot (\overleftrightarrow{\varepsilon} \cdot \nabla\phi) = 0 \qquad (1)$$

where $\overleftrightarrow{\varepsilon}$ is the permittivity dyadic. The boundary conditions at the interface of two media 1 and 2 are given by,

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0 \qquad (2a)$$

$$\hat{n} \cdot [(\sigma_1 \vec{E}_1 + j\omega \vec{D}_1) - (\sigma_2 \vec{E}_2 + j\omega \vec{D}_2)] = 0 \qquad (2b)$$

where $\vec{E}_{1,2} = -\nabla \phi_{1,2}$, $\vec{D}_{1,2} = \overleftrightarrow{\varepsilon}_{1,2} \cdot \vec{E}_{1,2}$; σ is the conductivity of the medium, and ω is the frequency. The two-dimensional boundary value problem associated with the evaluation of the quasi-TEM characteristics of the layered planar structures having, in general, lossy layers with a diagonal permittivity tensor can then be expressed as,

$$\varepsilon_x \frac{\partial^2 \phi}{\partial x^2} + \varepsilon_y \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (3a)$$

$\frac{\partial \phi}{\partial x}$ is continuous at the boundaries excluding the strips (3b)

$(\sigma + j\omega\, \varepsilon_y) \frac{\partial \phi}{\partial y}$ is continuous at the boundaries excluding the strips (3c)

and

$$\phi = V_i \;;\; i = 1,2,..N \text{ on the strips} \qquad (3d)$$

Expressing the above equations in a finite difference form [2,3] defines an electrical network analog having complex branches as shown in Fig. 1.

THE SOLUTION METHOD

For the case of infinitesimally thin strips on layered lossless media, Lennartson solved for the

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charge on each strip by first deriving the total resistance matrix representing the relationship between the node voltages and currents at the interface utilizing some basic transformations and properties of the electrical network. He then found the current in each strip which corresponds to the total charge on the strip by adding the currents on each node of the strip when a given potential is applied on all the strips. The procedure given in [1] provides a simple computational algorithm for obtaining the impedance matrix associated with the node voltages and currents at any interface. This procedure is readily modified to apply to the case of lossy, anisotropic layered structures with thin strips on one interface and to the case of thick strips at different boundaries as shown below.

The N_c voltages at a given boundary, where N_c represents the number of columns in the discretization scheme are expressed in terms of the corresponding node currents in the form of an $N_c \times N_c$ total impedance matrix as given by (Fig. 2),

$$V] = [Z]\ I] \tag{4}$$

We should note that in relation to the boundary value problem, the elements of [Z] are essentially a discrete representation of the boundary Green's function in real space. The elements of [Z] are obtained as in [1] in an algebraically equivalent transformed domain in terms of a diagonal matrix [Z] as given by

$$[\hat{Z}] = [A]\ [Z]\ [A] \tag{5}$$

where [A] is an involutory matrix consisting of the eigenvectors of the tridiagonal connection matrix at each level [1]. The diagonal matrix elements at the boundary level, L, are found from the recurrence relation

$$\hat{Z}_j^L = \frac{1}{\frac{1}{(\alpha_u^L)_j} + \frac{1}{(\alpha_\ell^L)_j} + \lambda_j y^L}, \quad j = 1,2,..N_c \tag{6}$$

where

$$(\alpha_{u,\ell}^{k+1})_j = \frac{1}{\frac{1}{(\alpha_{u,\ell}^k)_j} + \lambda_j y^k} + z^{k+1} \tag{7a}$$

with

$$\alpha_{u,\ell}^1 = z_{u,\ell}^1 \tag{7b}$$

$z_{u,\ell}^1$ is the impedance of the series element corresponding to the first level from the upper and the lower side, respectively. λ_j's are the eigenvalues of the connection matrix and are given by [1],

$$\lambda_j = 4 \sin^2 \left[\frac{j\pi}{2(N_c+1)}\right] \; ; \; j = 1,2,\ldots,N_c \tag{8}$$

In addition to the above straightforward modification to the method given in [1], we should note that for structures without a top or a bottom cover (grounded plane), an asymptotic expression can be derived for α_u or α_ℓ by requiring that,

$$(\alpha_{u,\ell}^{k+1})_j = (\alpha_{u,\ell}^k)_j \triangleq (\alpha_{u,\ell})_j; \; j = 1,2,..N_c \tag{9}$$

This simplifies the computations for open structures and structures without a ground plane. Equation (9), for an open structure, leads to:

$$(\alpha_u)_j = \frac{\lambda_j yz \pm [\lambda_j^2 y^2 z^2 + 4\lambda_j yz]^{1/2}}{2\lambda_j y} \tag{10}$$

where z and y are the impedance and the admittance elements of the homogeneous material.

THICK AND MULTILEVEL STRIP CASE

For structures with strips at more than one level or structures with thick strips, the above procedure is generalized in terms of the impedance matrix relating the voltages at the nodes of all the interfaces where the strips are located to the corresponding node currents. That is, the total impedance matrix is now of the order nN_c, where n is the number of different levels where the strips are located. However, the elements corresponding to self terms are evaluated in exactly the same manner as for the previous case and the mutual terms are easily derived in the transformed diagonalized domain. The transfer impedance term in this diagonalized domain relating the voltage on a given interface node at level k to the current on another interface node at level m in the same column, j, is found to be (Fig. 2b):

$$\hat{Z}_{km} = \left[\frac{1}{\frac{1}{(\alpha_u^k)_j} + \lambda_j y^k} \cdot \frac{\frac{1}{(\alpha_u^m)_j}}{\frac{1}{(\alpha_u^m)_j} + \frac{1}{(\alpha_\ell^m)_j} + \lambda_j y^m} \cdot \prod_{q=m+\mathrm{sgn}(k-m)}^{q=k-\mathrm{sgn}(k-m)} \frac{\frac{1}{(\alpha_u^q)_j}}{\frac{1}{(\alpha_u^q)_j} + \lambda_j y^q}\right] = \hat{Z}_{mk} \tag{11}$$

The transfer impedance matrix in real space relating voltages at level k and currents at level m is given by,

$$V]_k = [A]\ [\hat{Z}]_{km}\ [A]\ i]_m \triangleq [Z]_{km}\ I]_m \tag{12}$$

RESULTS

The propagation characteristics of several structures consisting of single and coupled lines on lossy, anisotropic, and layered structures have been computed by utilizing the above techniques. In order to check the accuracy of our calculations, we have computed the propagation characteristics of some uniaxial and lossy structures for which accurate, reliable results are available. The even and odd mode velocities calculated for some coupled microstrips on uniaxial substrates are given in Table I, together with the same values computed by Alexopoulos and Maas [15-Table I]. The results obtained for the attenuation constant of a microstrip line on silicon are shown in Fig. 3, together with the same results obtained by Simpson and Tseng [11]. Other results obtained for MIS lines are also found to be in good agreement with those in [8] and [13] for the range of conductivities in the lossy dielectric propagation region. Figure 4 shows the effect of the line thickness on the propagation characteristics of microstrips on lossy substrates. Figure 5 shows the microstrip parameters for an inverted microstrip studied by Spielman [1], together with his results. Our calculations for this case were conducted with a top cover which resulted in slightly higher values of the effective dielectric constant and the attenuation constant. Figure 6 shows the propagation characterstics of a simple symmetrical three line - two level structure chosen to demonstrate the application of this method to multilevel problems. Here the phase velocities of the three normal modes A (odd) B (even - even) and C (even - odd) are plotted as a function of the ratio of the thickness of the two dielectric layers.

We should mention that the typical computation time with a total of about 500 rows and 300 columns or interface nodes and 100 nodes on the metal strips on an HP 1000 model 65 is about 5 minutes and that the complexity of the configuration, including the number of columns, is only limited by the storage and the speed of the computer. Also the method can be extended to three-dimensional layered medium problems including resonators and lumped elements for MIC's.

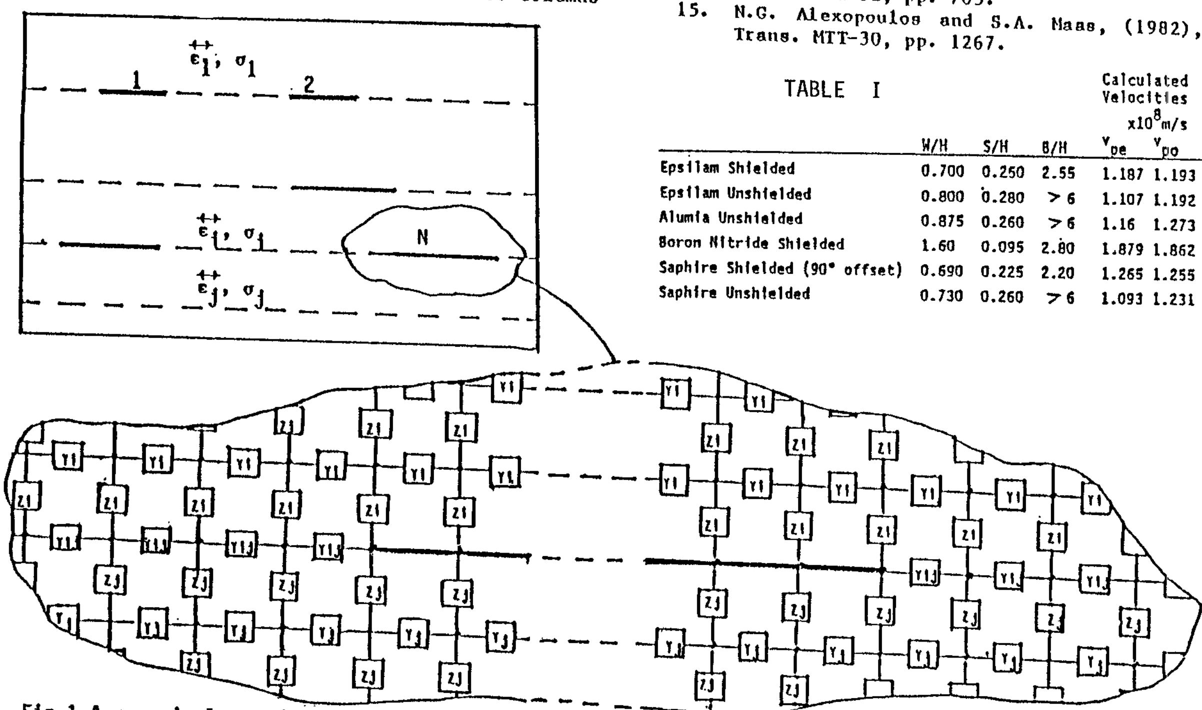

Fig.1 A generic layered structure with N strips together with the network analog for a representive region. Y's and Z's depend on the structure,e.g.,
For lossy isotropic medium : $Y_i = \sigma_i + j\,\omega\epsilon_i$; $Z_i = 1/Y_i$; $Y_{ij} = \frac{1}{2}(Y_i + Y_j)$
For lossless anisotropic medium: $Y_i = j\omega\,\epsilon_{xi}$; $Z_i = 1/j\,\omega\epsilon_{yi}$; $Y_{ij} = \frac{1}{2}(Y_i + Y_j)$

REFERENCES

1. B.L. Lennartson, (1972), IEEE Trans. MTT-20; pp. 582.
2. G. Liebmann, (1950), Brit. Jour. Appl. Physics, Vol. 1, pp. 92.
3. M.V. Schneider, (1965), IEEE Trans. MTT, Vol. 13, pp. 793.
4. U. Schulz, R. Pregla, (1980), Arch. Elect. u Ubertrag 34, pp. 169.
5. S.B. Worm, R. Pregla, (1984), IEEE Trans. MTT, Vol. 32, pp. 191.
6. R. Pregla, S.B. Worm, (1984), IEEE-S-MTT Dig., pp. 348
7. T.M. Hyltin, (1965), IEEE Trans. MTT-13; pp. 777.
8. H. Hasequawa, et al., (1971), IEEE Trans. MTT-19; pp. 869.
9. R.A. Pucel, et al., (1968), IEEE Trans. MTT-16; pp. 342.
10. M.V. Schneider, (1959), BSTJ-48, pp. 1421 and pp. 2325.
11. T.L. Simpson and B. Tseng, (1976), IEEE Trans. MTT-24, pp. 106.
12. B.E. Spielman, (1977), IEEE Trans. MTT-25, pp. 748.
13. G.W. Hughes and R.M. White, (1975), IEEE Trans. ED-22, pp. 945.
14. R.F. Harrington and C. Wei, (1984), IEEE Trans. MTT-32, pp. 705.
15. N.G. Alexopoulos and S.A. Maas, (1982), IEEE Trans. MTT-30, pp. 1267.

TABLE I

	W/H	S/H	B/H	Calculated Velocities $\times 10^8$ m/s v_{pe}	v_{po}	From [10] $\times 10^8$ m/s v_{pe}	v_{po}
Epsilam Shielded	0.700	0.250	2.55	1.187	1.193	1.207	1.210
Epsilam Unshielded	0.800	0.280	> 6	1.107	1.192	1.138	1.204
Alumia Unshielded	0.875	0.260	> 6	1.16	1.273	1.15	1.286
Boron Nitride Shielded	1.60	0.095	2.80	1.879	1.862	1.876	1.875
Saphire Shielded (90° offset)	0.690	0.225	2.20	1.265	1.255	1.256	1.257
Saphire Unshielded	0.730	0.260	> 6	1.093	1.231	1.086	1.227

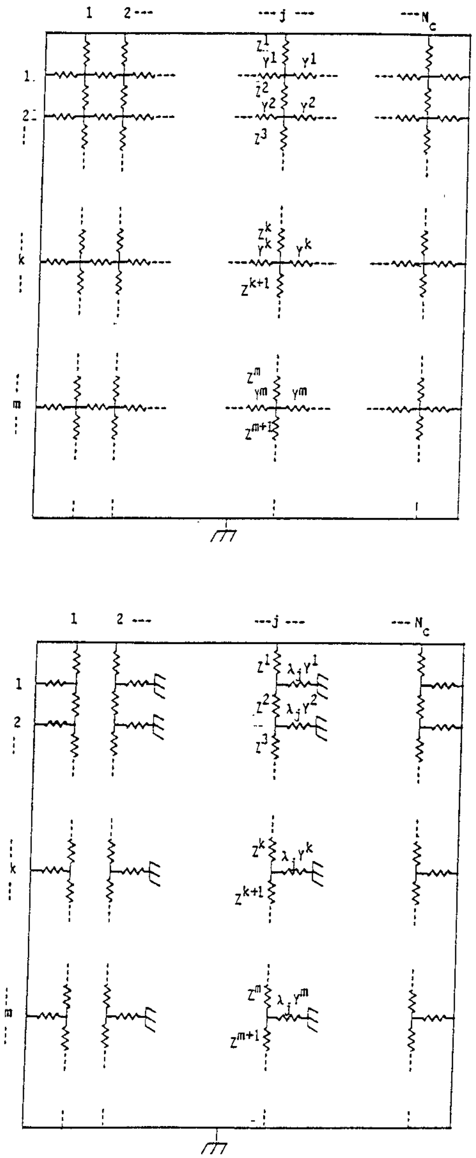

Fig. 2. The discrete network analog and its algebraically equivalent transformed network.

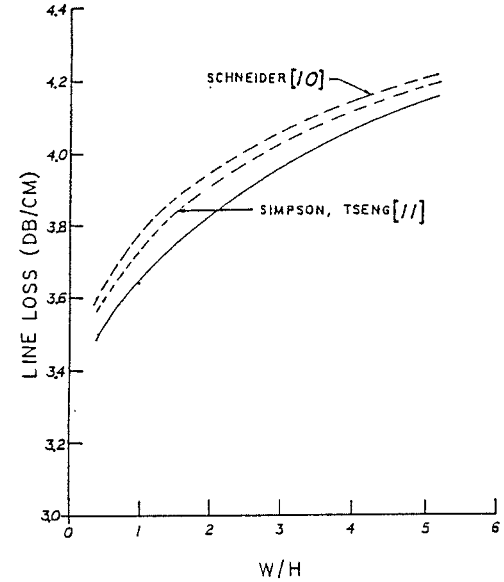

Fig. 3. Dielectric loss for microstrips on silicon.

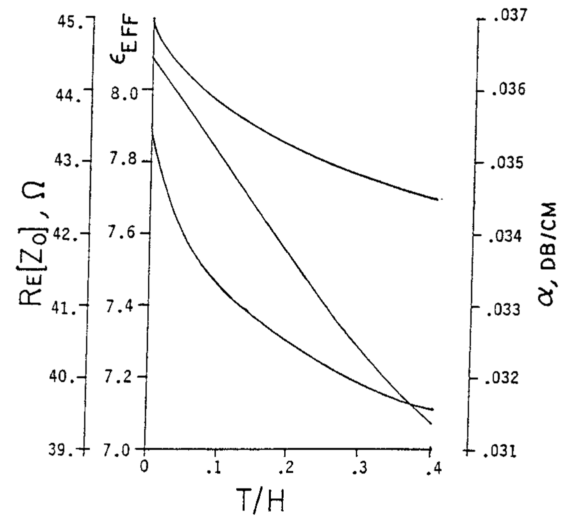

Fig. 4. Propagation characteristics as a function of microstrip thickness. Substrate silicon, σ =0.01$\mho$/m, W=H.

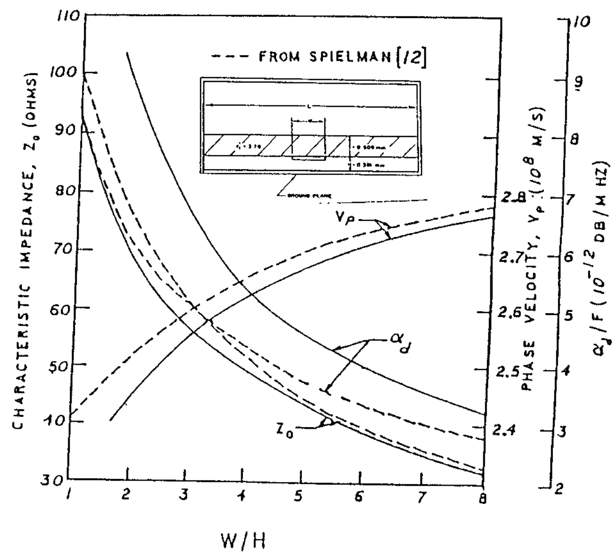

Fig. 5. Properties of inverted micro-strip line.

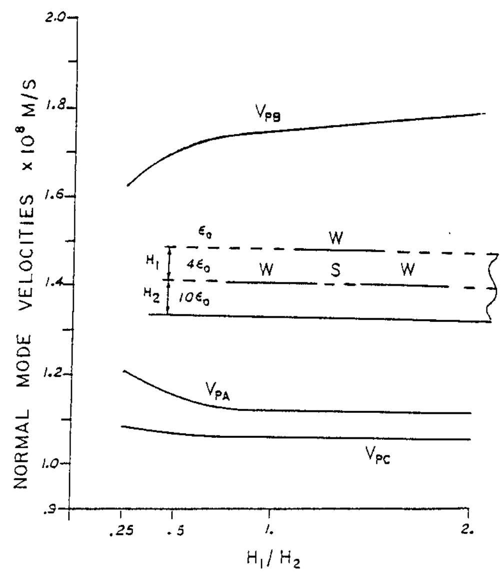

Fig. 6. Normal mode velocities of a two level-three line structure. W=2S=H_2.